Tensor Codes

Eimear Byrne

University College Dublin

Connections between codes and tensors have been known since the 1970s. In particular, algebraic coding theory can play a role in estimating bilinear complexity [1]. More recently, new connections between codes tensors have been observed. Matrix codes for the rank metric have a natural description in terms of 3-tensors. Encoding and decoding such codes is facilitated by notions of generator and parity-check tensors [3, 4]. Expressing a generator tensor as a minimal sum of rank-1 tensors gives a compact representation of a code and hence tensor rank is a highly relevant parameter of matrix codes.

Subspaces of higher-order tensors themselves form an interesting class of codes, called tensor codes, for which the ambient space is endowed with a distance function arising from tensor rank. Roth was the first to study these and introduced a class of tensor codes as a generalisation of the well-known class of Delsarte-Gabidulin-Roth (Gabidulin) MRD codes [5].

In this talk we will give an introduction to tensor codes and their properties. We will describe some classes of support lattices associated with tensor codes and explain how these lead to different generalised tensor code weights. Such weights can be used to identify different classes of extremal codes. They also act as distinguishers of inequivalent tensor codes. We will mention some connections to other combinatorial objects and give some open problems in this area.

Bibliography

1. R.W. Brockett, D. Dobkin, “On the optimal evaluation of a set of bilinear forms,” Linear Algebra Appl. 19 (3), 1978.
2. E. Byrne, G. Cotardo, “Tensor Codes and Their Invariants,” SIAM. J. Discrete Math., 37 (3), 2023.
3. E. Byrne, A. Neri, A. Ravagnani, J. Sheekey, “Tensor Representation of Rank-Metric Codes,” SIAM J. Appl. Algebra Geom., 3, 2019.
4. E. Franch, P. Gaborit, C. Li, “Generalized low rank parity check codes,ÏEEE Information Theory Workshop, ITW 2023, Saint-Malo, France, April 23-28, 2023.
5. R. M. Roth, “Tensor codes for the rank metric,” IEEE Transactions on Information Theory, 42 (6), 1996.