Radboud University

Security of Encryption Modes and an Exposition of Proof Techniques

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Keyed Symmetric Cryptography

General Setting

- Two parties, Alice and Bob, communicate over a public channel
  - They have agreed on a joint key and use it to transmit data

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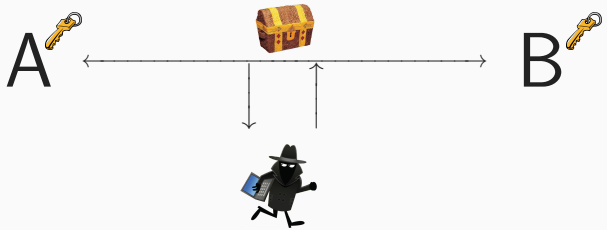

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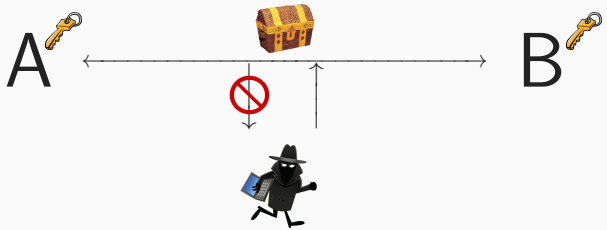

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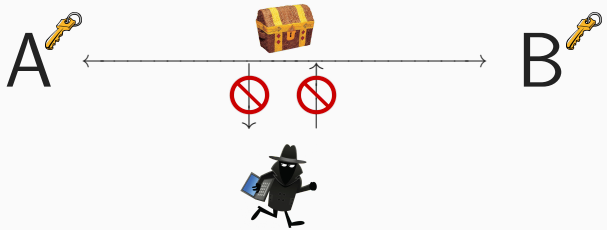

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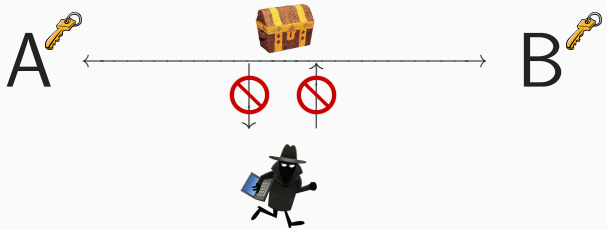

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In this presentation I will focus on confidentiality

One-Time Pad Encryption

Encryption:

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Stream Ciphers

- Generate long keystream Z from short key K
- Much more practical!
- Security degrades:
  1. Key guessing still succeeds with probability $1/2^{|K|}$ but now with shorter key
  2. The stream cipher mechanism is another focal point of attack

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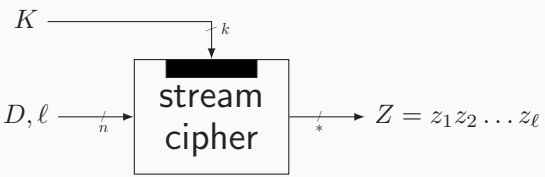
We need something more sophisticated!

How to Model Security?

Modern Stream Ciphers



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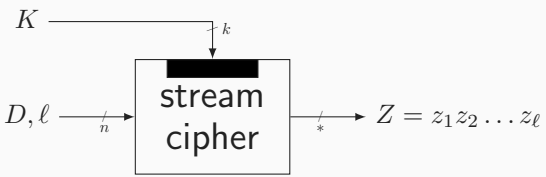

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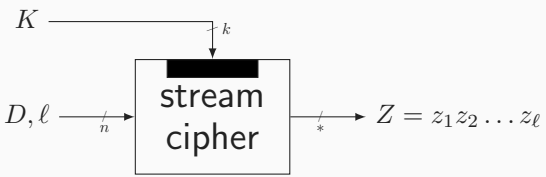

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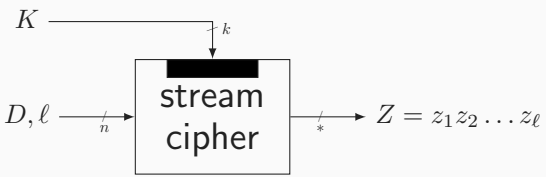
- Example: data streams, e.g., pay TV and telephone, often split data in relatively short, numbered, frames. The frame number may serve as diversifier:

$$C_i = M_i \oplus \mathsf{SC}(K, i, |M_i|)$$



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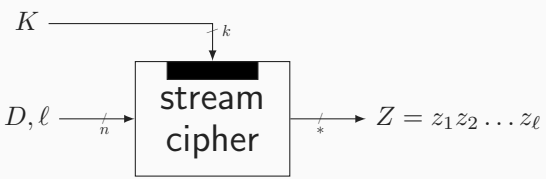
When is a stream cipher strong enough?

Stream Cipher Security, Intuition (1/3)



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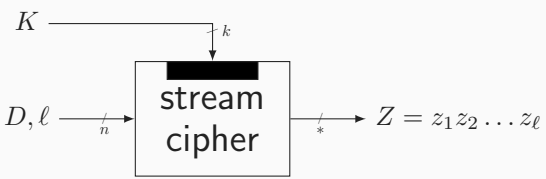

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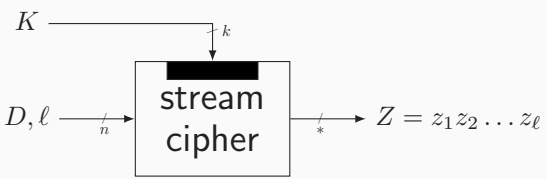

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- SC_K should behave like a random oracle

Intermezzo: Random Oracle

Random Oracle

- A database of input-output tuples
- Initially empty

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...	...
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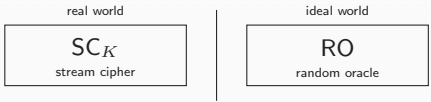
    - update (D, Z) in the list

Stream Cipher Security, Intuition (2/3)



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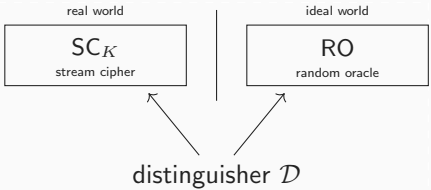

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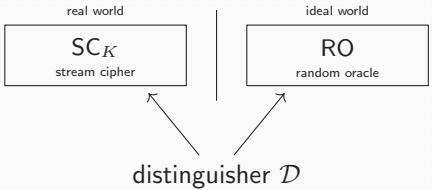

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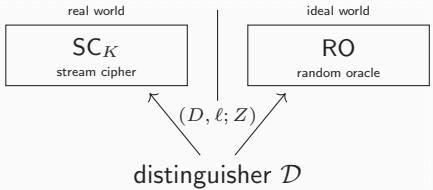

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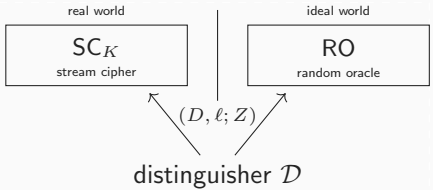

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  - At the end, $\mathcal{D}$ has to guess the outcome of the toss coin (head/tail)

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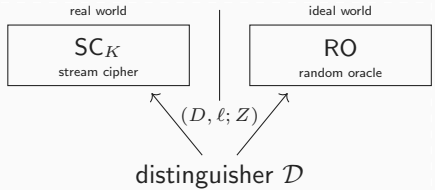

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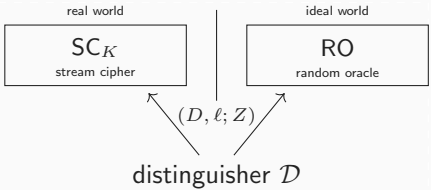

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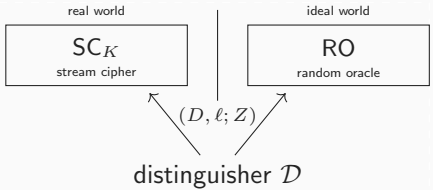

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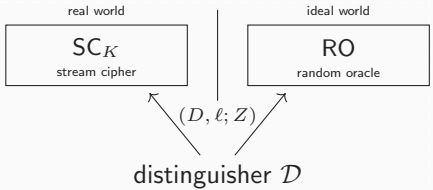

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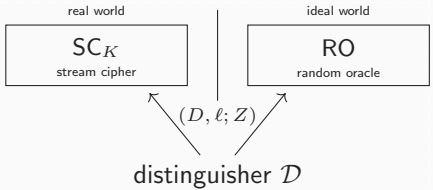
- $\mathcal{D}$ is limited by certain constraints

Stream Cipher Security, Intuition (3/3)

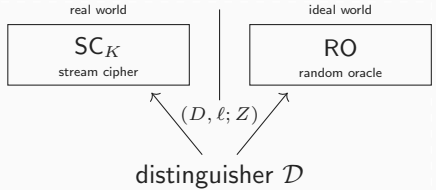


- Denote $\mathcal{D}$'s success probability in correctly guessing head/tail by $\mathbf{Pr}\,(\text{success})$
- $\mathcal{D}$ can always guess and succeeds with probability $\geq 1/2$, so we scale the probability to $\mathcal{D}$'s advantage:

$$\begin{aligned}\mathbf{Adv}(\mathcal{D}) &= 2 \cdot \mathbf{Pr}\,(\text{success}) - 1 \\ &= \mathbf{Pr}\left(\mathcal{D}^{\mathsf{SC}_K} \text{ returns head}\right) - \mathbf{Pr}\left(\mathcal{D}^{\mathsf{RO}} \text{ returns head}\right)\end{aligned}$$

- $\mathcal{D}$ is limited by certain constraints
  - Data (or online) complexity q: total cost of queries $\mathcal{D}$ can make
  - Computation (or time) complexity t: everything that $\mathcal{D}$ can do "on its own"

Stream Cipher Security, Formal



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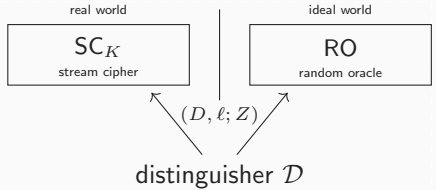

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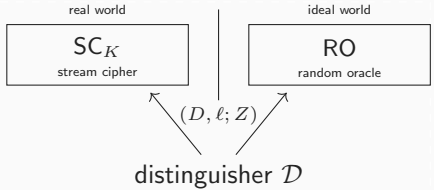

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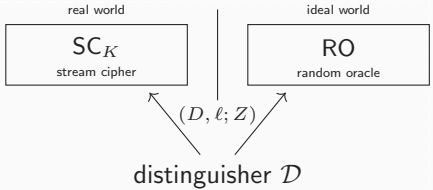

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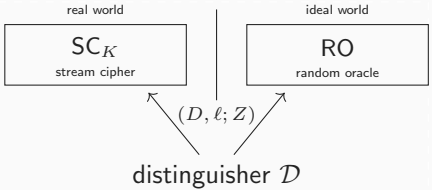

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- $\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{SC}}(q, t)$: maximum advantage over any distinguisher with complexity q, t

Generic Stream Cipher Design

Generic Stream Cipher Design (1/2)

- Classical approach: LFSRs strengthened with non-linear component
- Modern approach: building construction from smaller cryptographic primitive

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- Classical approach: LFSRs strengthened with non-linear component
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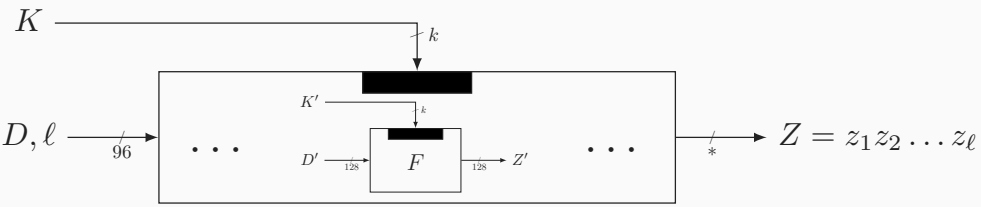
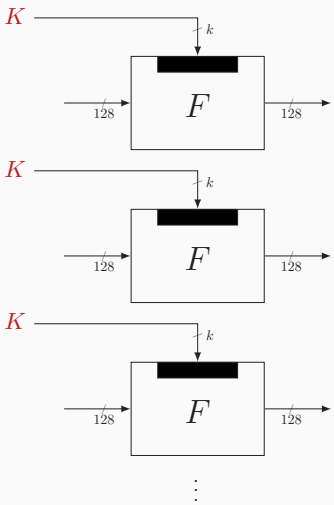
- Suppose (for the sake of argument):
  - we **know** how to build a strong stream cipher F with fixed-length output
  - we **want** to build a stream cipher with variable-length output



Generic Stream Cipher Design (2/2)

Design

- Feed K to primitive



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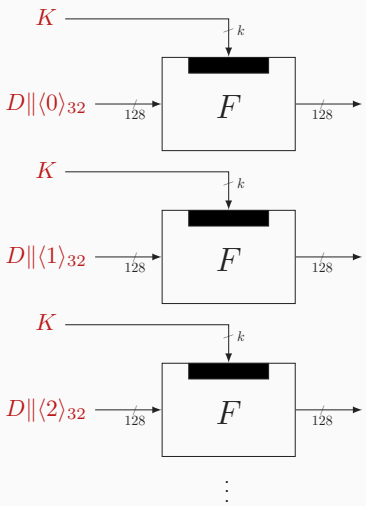

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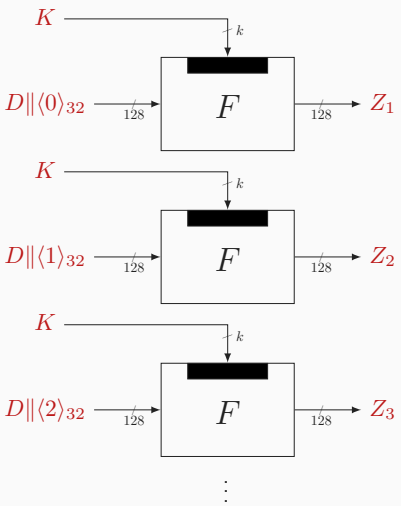

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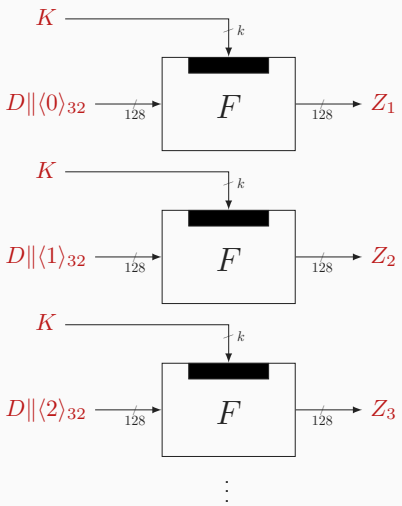

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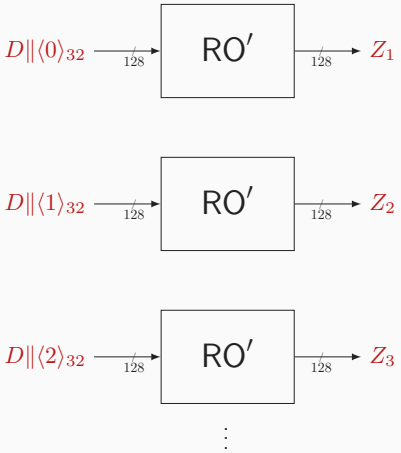

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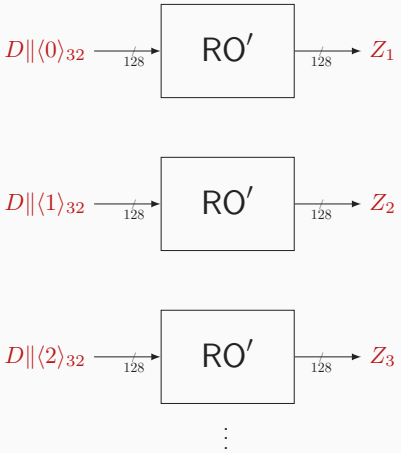

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- If F_K is hard to distinguish from a RO′
- Then construction is hard to distinguish from RO

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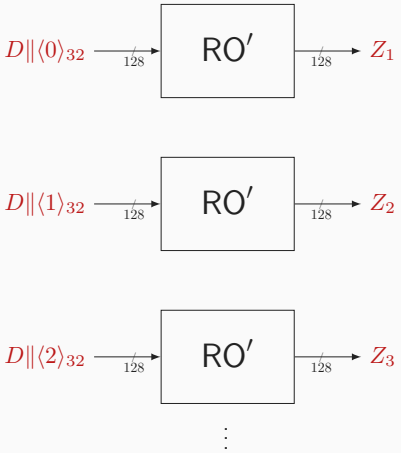

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Generic Stream Cipher Design (2/2)

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- Evaluate primitive as
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- Concatenate outputs:

$$Z = Z_1$$

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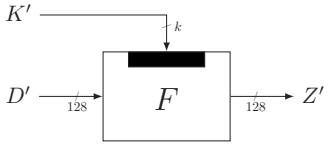
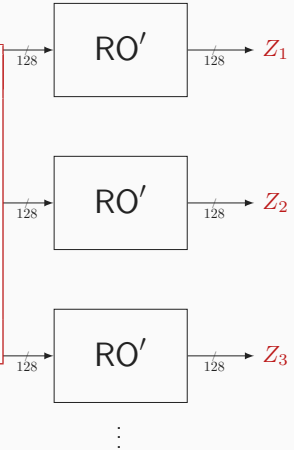
Unfortunately, we do not know how to easily construct a function



that behaves like a RO′



Block Ciphers

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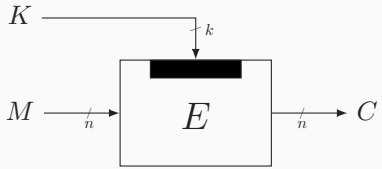

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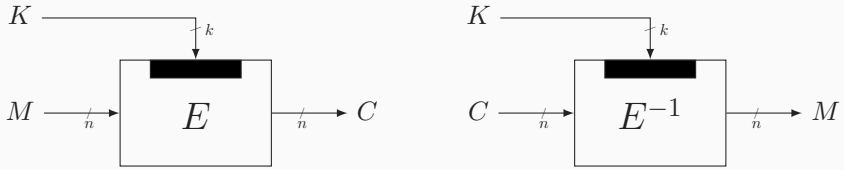

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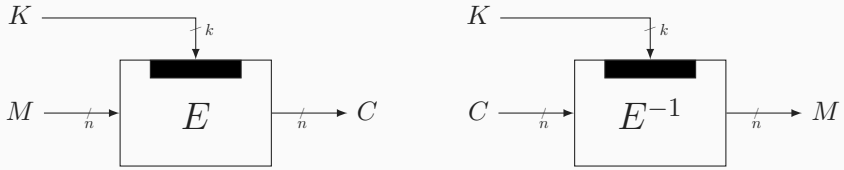

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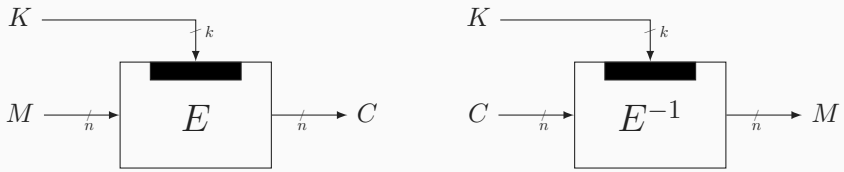

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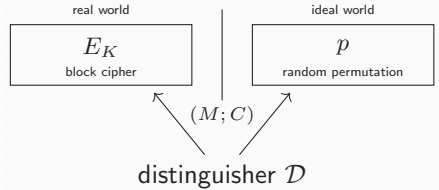
- A good block cipher should behave like a random permutation

Block Cipher Security



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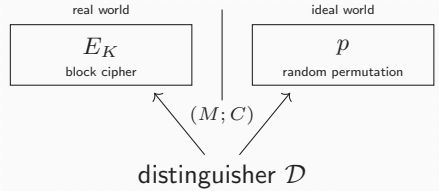

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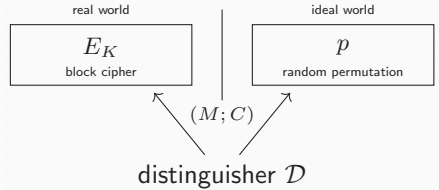

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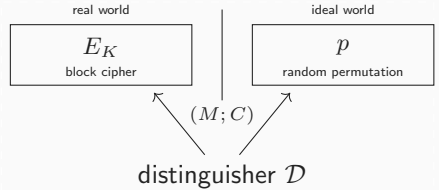

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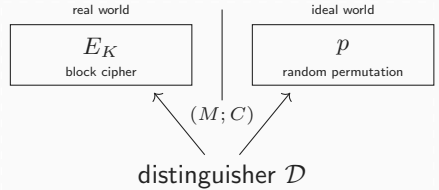

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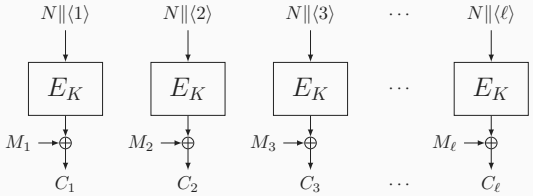
Counter Mode Encryption

Counter (CTR) Mode



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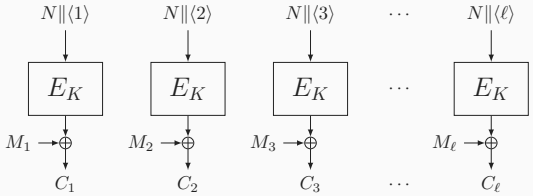

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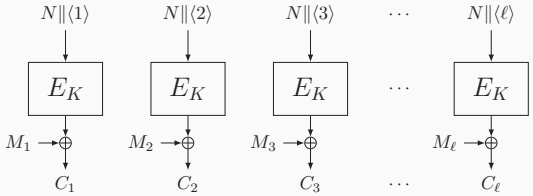

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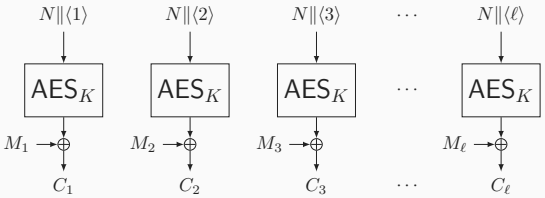
- Let us investigate that!

Security of Counter Mode Based on AES

- Let us consider counter mode based on AES: CTR[AES_K]



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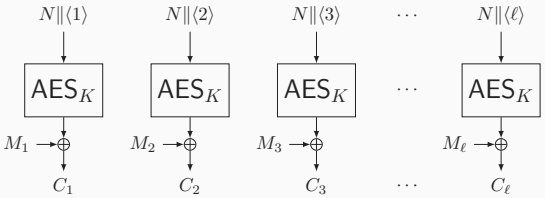

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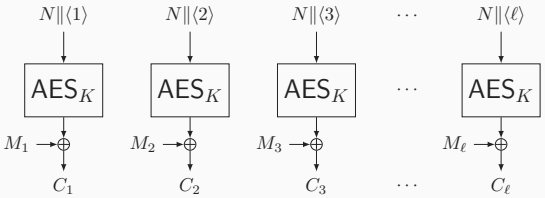

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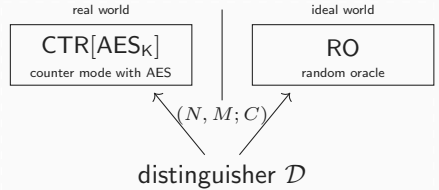
  - AES itself is sufficiently secure: $\mathbf{Adv}^{\mathrm{prp}}_{\mathsf{AES}}(q,t)$ is small

Security of Counter Mode Based on AES: Model



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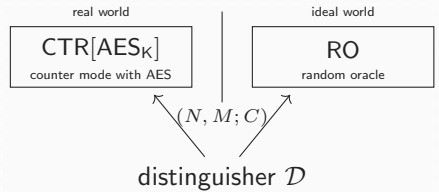

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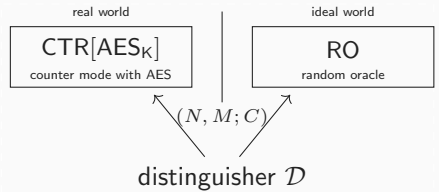

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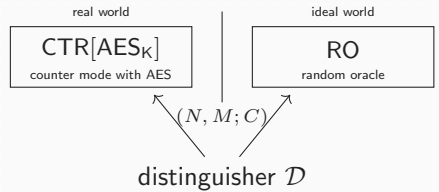

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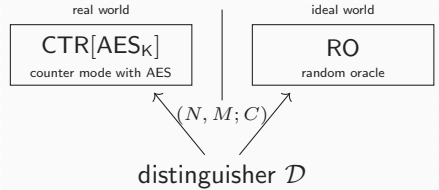

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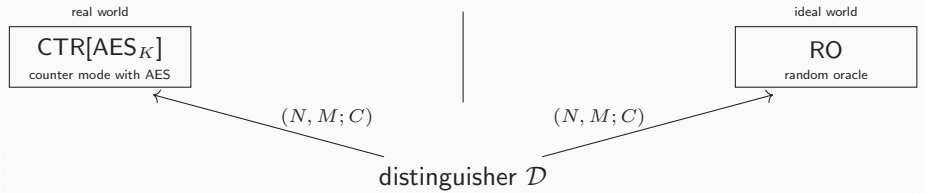
- $\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR[AES]}}(q, t)$: maximum advantage over any $\mathcal{D}$ with q/t blocks/time

Proof: Overview



- For any (fixed) distinguisher $\mathcal{D}$ (later, we supremize over all), we have to bound:

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR[AES]}}(\mathcal{D}) = \Delta_{\mathcal{D}}\left(\mathsf{CTR[AES}_K\mathsf{]}\,;\,\mathsf{RO}\right) = \left|\mathbf{Pr}\left(\mathcal{D}^{\mathsf{CTR[AES}_K\mathsf{]}} = 1\right) - \mathbf{Pr}\left(\mathcal{D}^{\mathsf{RO}} = 1\right)\right|$$

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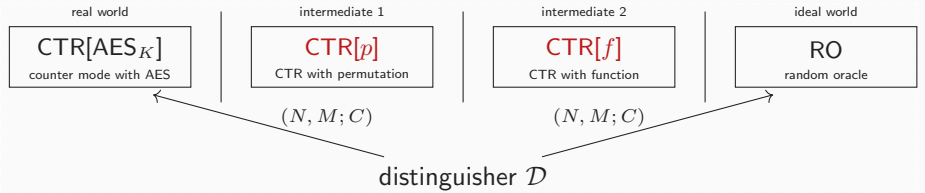

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- We add intermediate worlds CTR$[p]$ and CTR$[f]$ for random p and f

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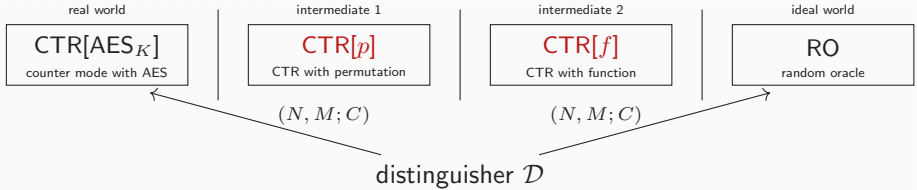

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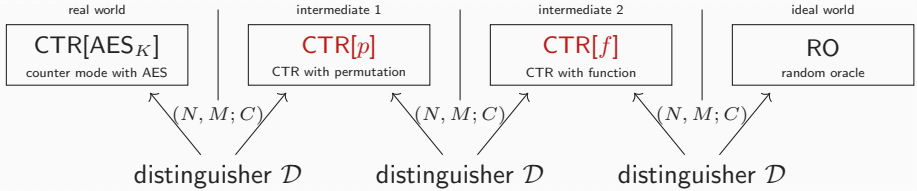
- By the triangle inequality:

$$\Delta_{\mathcal{D}}\left(\mathsf{CTR[AES}_K\mathsf{]}\,;\,\mathsf{RO}\right) \leq \Delta_{\mathcal{D}}\left(\mathsf{CTR[AES}_K\mathsf{]}\,;\,\mathsf{CTR}[p]\right) + \Delta_{\mathcal{D}}\left(\mathsf{CTR}[p]\,;\,\mathsf{CTR}[f]\right) + \Delta_{\mathcal{D}}\left(\mathsf{CTR}[f]\,;\,\mathsf{RO}\right)$$

Proof: Overview



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- $\mathcal{D}$'s goal: distinguish CTR[AES_K] from CTR[p]

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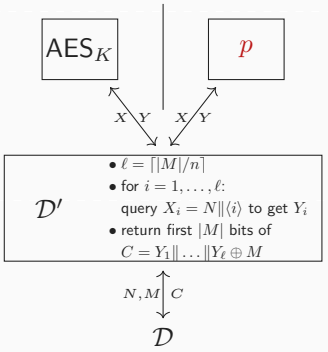

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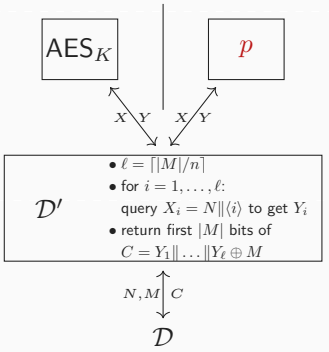

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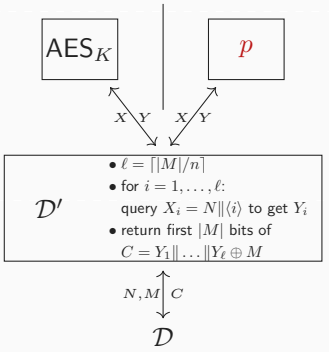
- But we have seen this distance before:
  $\Delta_{\mathcal{D}'}\,(\text{AES}_K\ ;\ p) = \mathbf{Adv}^{\text{prp}}_{\text{AES}}(\mathcal{D}') \leq \mathbf{Adv}^{\text{prp}}_{\text{AES}}(q, t')$
  (t' slightly larger than t)



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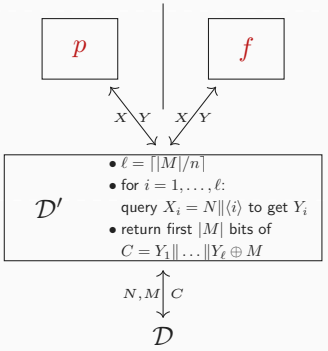

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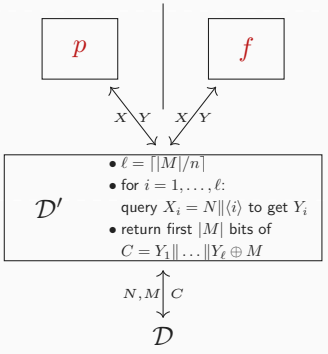

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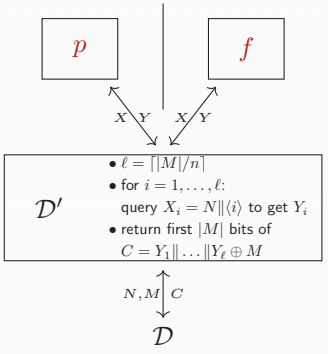
- This is a well-known distance, called the RP-RF switch



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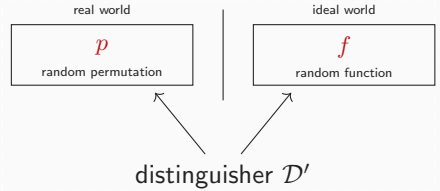

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  - real world: without replacement
  - ideal world: with replacement

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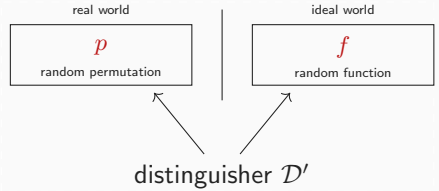

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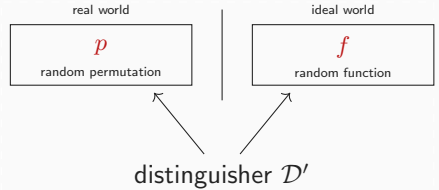

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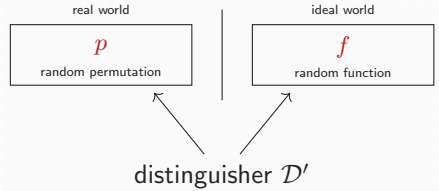

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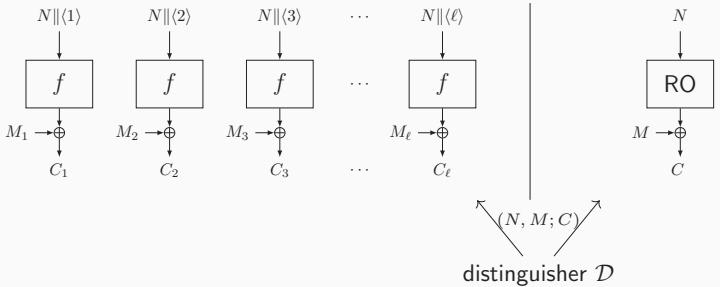
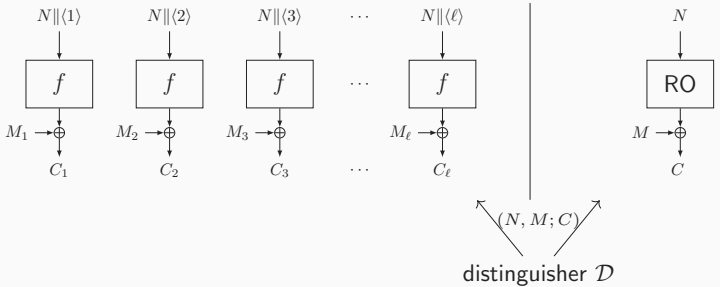
- Hence: $\Delta_{\mathcal{D}'}(p\,;\,f) \leq \binom{q}{2}/2^n$

Proof: From CTR[f] to RO



- In real world: f is a random function that is never evaluated for repeated $N\|\langle i\rangle$
- In ideal world: RO is a random oracle that is never evaluated for repeated N

Proof: From CTR[f] to RO



- In real world: f is a random function that is never evaluated for repeated $N\|\langle i\rangle$
- In ideal world: RO is a random oracle that is never evaluated for repeated N
- Hence: $\Delta_{\mathcal{D}}\left(\mathsf{CTR}[f]\ ;\ \mathsf{RO}\right) = 0$

Proof: Conclusion

- Recall goal: bounding $\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR[AES]}}(\mathcal{D})$ for any $\mathcal{D}$ querying q blocks in t time

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$$\begin{aligned}\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR[AES]}}(\mathcal{D}) &= \Delta_{\mathcal{D}}\left(\mathsf{CTR}[\mathsf{AES}_K]\ ;\ \mathsf{RO}\right)\\ &\leq \Delta_{\mathcal{D}}\left(\mathsf{CTR}[\mathsf{AES}_K]\ ;\ \mathsf{CTR}[p]\right) + \Delta_{\mathcal{D}}\left(\mathsf{CTR}[p]\ ;\ \mathsf{CTR}[f]\right) + \Delta_{\mathcal{D}}\left(\mathsf{CTR}[f]\ ;\ \mathsf{RO}\right)\\ &\leq \mathbf{Adv}^{\mathrm{prp}}_{\mathsf{AES}}(q, t') + \binom{q}{2}/2^n + 0\end{aligned}$$

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- As this reasoning holds for all distinguishers $\mathcal{D}$ querying q blocks in t time, we obtain:

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR[AES]}}(q,t) \leq \mathbf{Adv}^{\mathrm{prp}}_{\mathsf{AES}}(q,t') + \binom{q}{2}/2^n$$

Beyond Birthday Bound Security

Birthday Paradox

For a random selection of 23 people, with a probability at least 50% two of them share the same birthday

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General Birthday Paradox

- Consider space $\mathcal{S} = \{0,1\}^n$
- Randomly draw q elements from $\mathcal{S}$
- Expected number of collisions:

$$\mathbf{Ex}\left[\text{collisions}\right] = \binom{q}{2}/2^n$$

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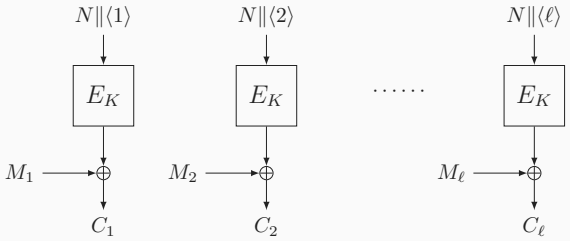
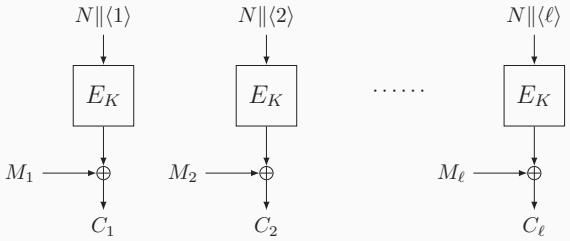
- Important phenomenon in cryptography

Counter Mode Based on Pseudorandom Permutation



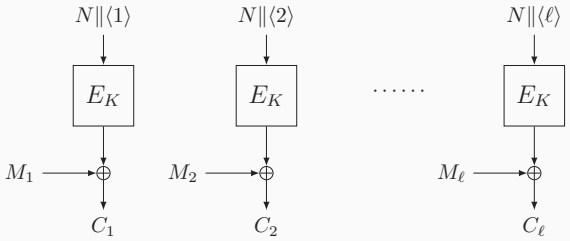
Counter Mode Based on Pseudorandom Permutation



- Security bound:

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR}[E]}(q,t) \leq \mathbf{Adv}^{\mathrm{prp}}_{E}(q,t') + \binom{q}{2}/2^n$$

Counter Mode Based on Pseudorandom Permutation

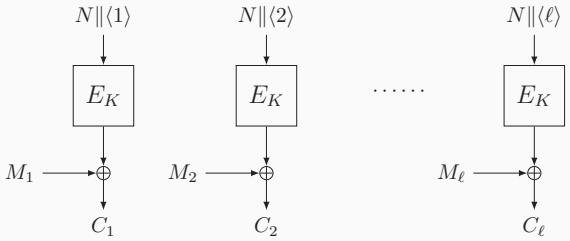


- Security bound:

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR}[E]}(q,t) \leq \mathbf{Adv}^{\mathrm{prp}}_{E}(q,t') + \binom{q}{2}/2^n$$

- CTR$[E]$ is secure as long as:
  - E_K is a secure PRP
  - Number of encrypted blocks $q \ll 2^{n/2}$

Counter Mode Based on Pseudorandom Permutation



- $M_i \oplus C_i$ is distinct for all q blocks
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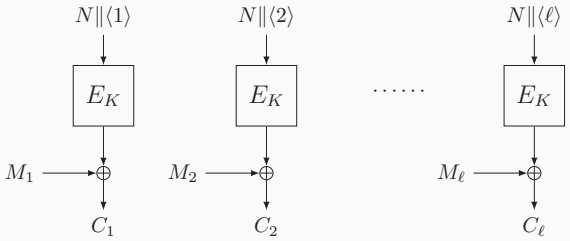
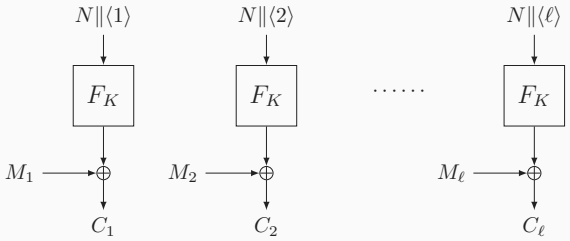


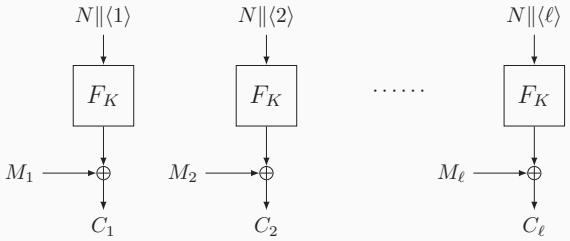
- $M_i \oplus C_i$ is distinct for all q blocks
- Unlikely to happen for random string
- Distinguishing attack in $q \approx 2^{n/2}$ blocks:

$$\binom{q}{2}/2^n \lesssim \mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR}[E]}(q,t)$$

Counter Mode Based on Pseudorandom Function



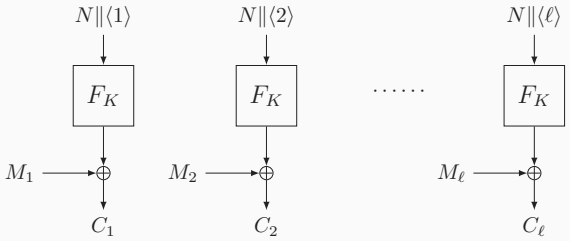
Counter Mode Based on Pseudorandom Function



- Security bound:

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR}[F]}(q,t) \leq \mathbf{Adv}^{\mathrm{prf}}_{F}(q,t')$$

Counter Mode Based on Pseudorandom Function

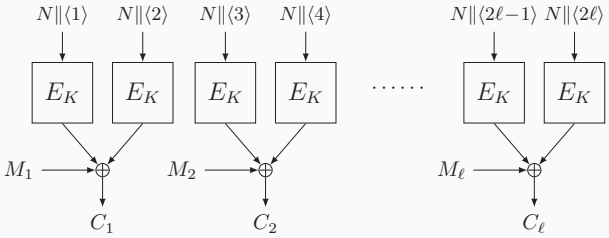


- Security bound:

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR}[F]}(q,t) \leq \mathbf{Adv}^{\mathrm{prf}}_{F}(q,t')$$

- CTR$[F]$ is secure as long as F_K is a secure PRF
- Birthday bound security loss disappeared

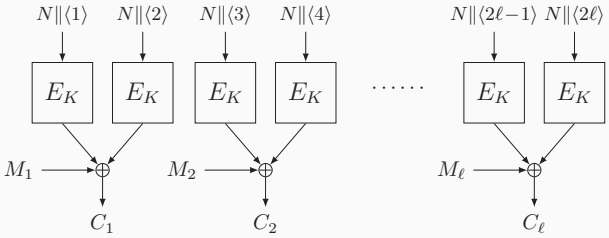
Counter Mode Based on XoP



- Security bound [Pat08, DHT17]:

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR[XoP]}}(q,t) \leq \mathbf{Adv}^{\mathrm{prf}}_{\mathsf{XoP}}(q,t')$$

Counter Mode Based on XoP



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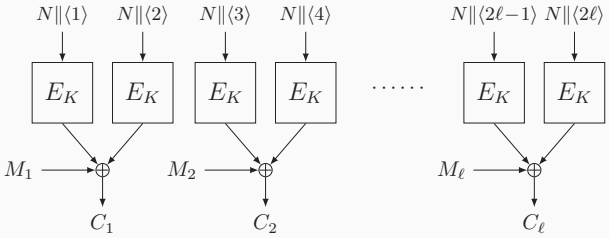
Counter Mode Based on XoP



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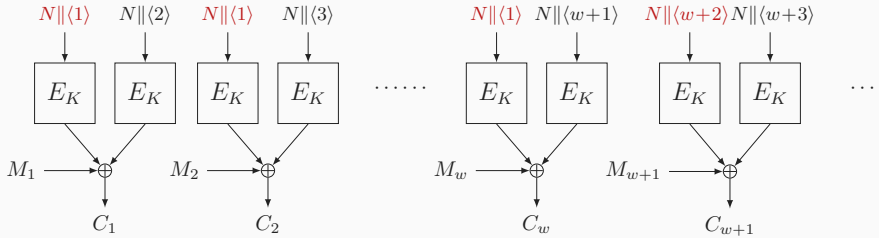
- Beyond birthday bound but 2x as expensive as $\mathsf{CTR}[E]$

CENC by Iwata [Iwa06]



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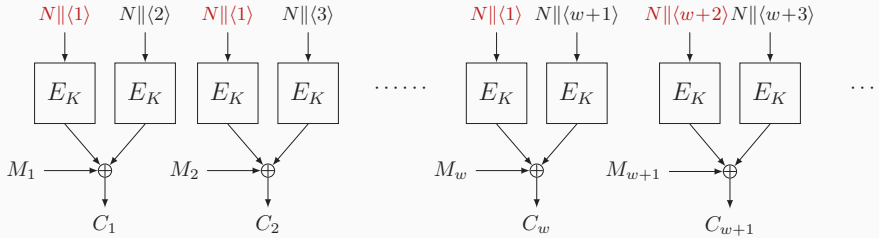

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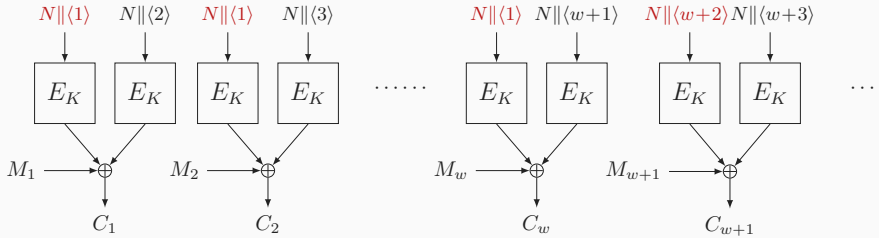

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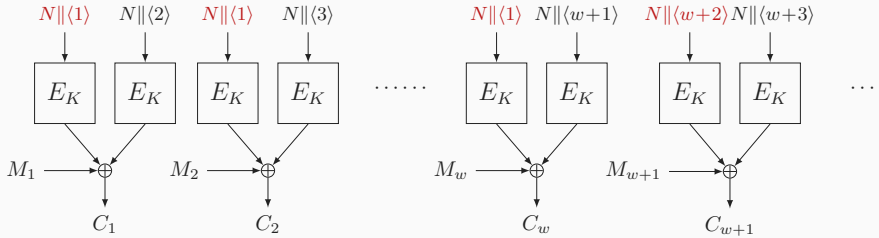

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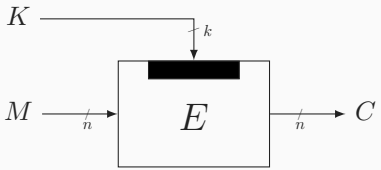
- Security of XoP and XoP$[w]$ can be proven using mirror theory [Pat03]

Accordion Modes

Block Ciphers



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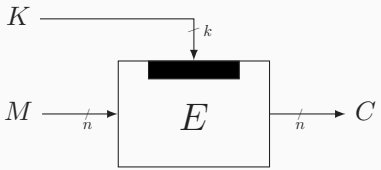
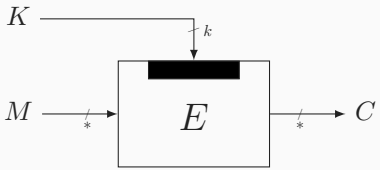
Block Ciphers



- Message M encrypted to ciphertext C with secret key K
- Fixed block size
- In order to encrypt variable sized messages, we need a mode of operation
  - These modes require a nonce

Wide Block Ciphers



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- A wide block cipher is a block cipher with a variable block size

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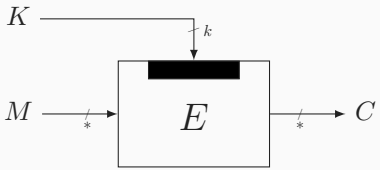
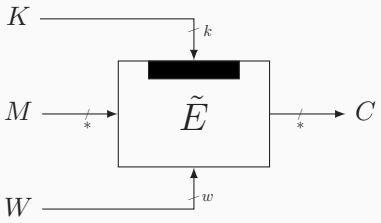


- Alternatively, we can design a wide block cipher
- A wide block cipher is a block cipher with a variable block size
- Every part of the output (ideally) depends on every part of the input

Tweakable Wide Block Ciphers



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- Tweak W public, ciphertext completely changes with a different tweak

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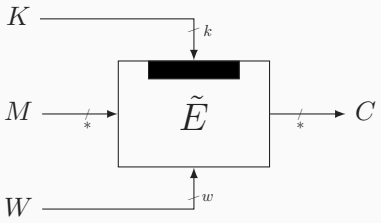

- A tweakable wide block cipher additionally has a tweak
- Tweak W public, ciphertext completely changes with a different tweak
- Useful for e.g. disk encryption, where every sector gets its own tweak

NIST's Incentive to Develop Accordion Mode

- March 2024: NIST announced quest for tweakable wide block ciphers
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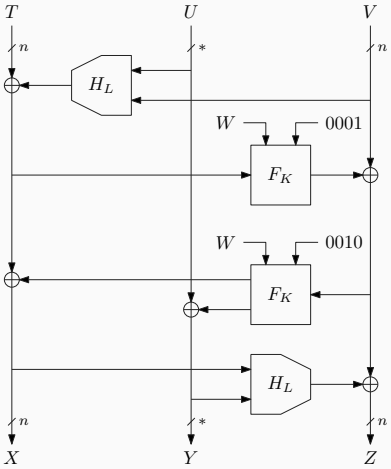
Now: high-level idea of our recent proposals

Docked Double Decker [GDM19]



Building Blocks

- F_K: stream cipher
- H_L: universal hash

Construction

- Feistel-like structure
- Outer lanes of fixed size
- Inner lane of variable size

Goals

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  - AES [DR02, DR20]
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Goals and Hurdles

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Hurdles

- AES is not a tweakable blockcipher
- AES is rather small (circular reasoning?)
- AES in typical stream cipher modes only gives birthday bound security

Efficient Instantiations of Docked Double Decker [DMMT24]

ddd-AES

- H_L instantiated using Polyval (as in GCM-SIV)
- F_K instantiated as variant of CTR: tweak used to randomize inputs to AES_K

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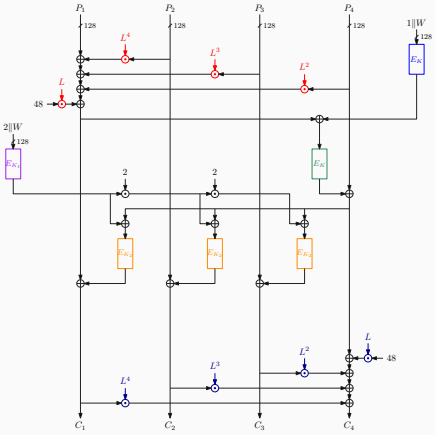
Instantiations turn out to be very competitive and well parallelizable

Implementation Design of $ddd\text{-}AES$ (512-Bit Message)



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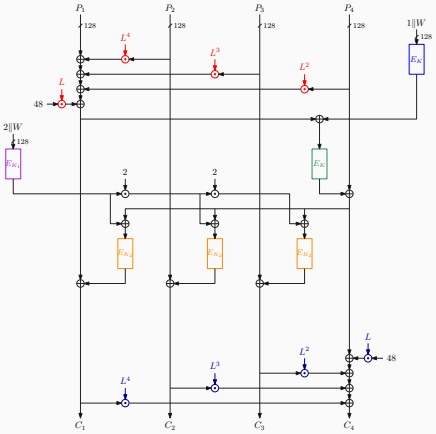

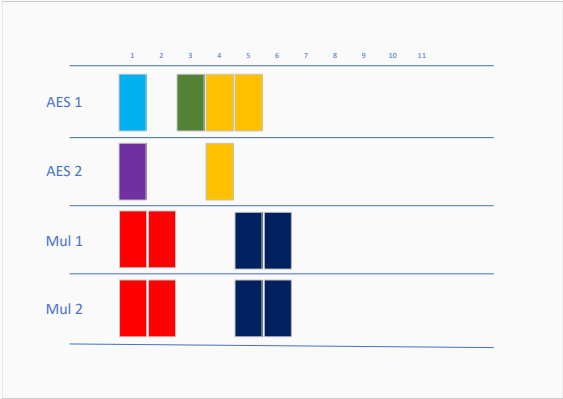

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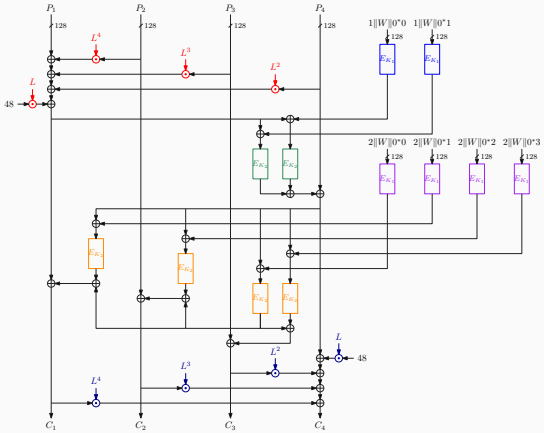

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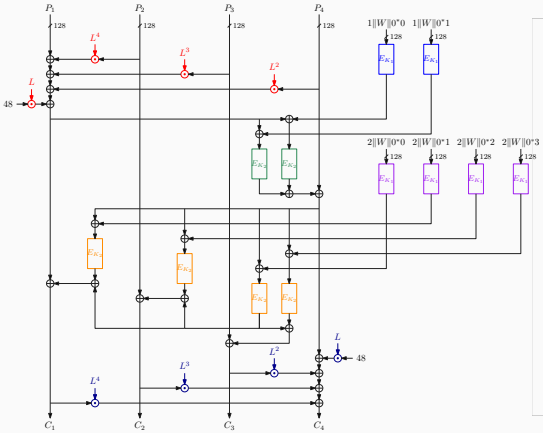

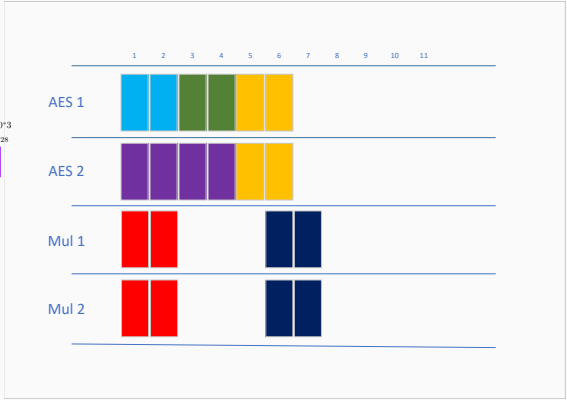

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Thank you for your attention!

References i

Wei Dai, Viet Tung Hoang, and Stefano Tessaro.
Information-Theoretic Indistinguishability via the Chi-Squared Method.
In Jonathan Katz and Hovav Shacham, editors, *Advances in Cryptology - CRYPTO 2017 - 37th Annual International Cryptology Conference, Santa Barbara, CA, USA, August 20-24, 2017, Proceedings, Part III*, volume 10403 of *Lecture Notes in Computer Science*, pages 497–523. Springer, 2017.

Christoph Dobraunig, Krystian Matusiewicz, Bart Mennink, and Alexander Tereschenko.
Efficient Instances of Docked Double Decker With AES, and Application to Authenticated Encryption.
Cryptology ePrint Archive, Report 2024/084, 2024.
https://eprint.iacr.org/2024/084.

References ii


Joan Daemen and Vincent Rijmen.
The Design of Rijndael: AES - The Advanced Encryption Standard.
Information Security and Cryptography. Springer, 2002.


Joan Daemen and Vincent Rijmen.
The Design of Rijndael - The Advanced Encryption Standard (AES), Second Edition.
Information Security and Cryptography. Springer, 2020.


Aldo Gunsing, Joan Daemen, and Bart Mennink.
Deck-Based Wide Block Cipher Modes and an Exposition of the Blinded Keyed Hashing Model.
IACR Trans. Symmetric Cryptol., 2019(4):1–22, 2019.

References iii

Tetsu Iwata, Bart Mennink, and Damian Vizár.
CENC is Optimally Secure.
Cryptology ePrint Archive, Report 2016/1087, 2016.
http://eprint.iacr.org/2016/1087.

Tetsu Iwata.
New Blockcipher Modes of Operation with Beyond the Birthday Bound Security.
In Matthew J. B. Robshaw, editor, *Fast Software Encryption, 13th International Workshop, FSE 2006, Graz, Austria, March 15-17, 2006, Revised Selected Papers*, volume 4047 of *Lecture Notes in Computer Science*, pages 310–327. Springer, 2006.


David A. McGrew and John Viega.
The Security and Performance of the Galois/Counter Mode (GCM) of Operation.
In Anne Canteaut and Kapalee Viswanathan, editors, *Progress in Cryptology - INDOCRYPT 2004, 5th International Conference on Cryptology in India, Chennai, India, December 20-22, 2004, Proceedings*, volume 3348 of *Lecture Notes in Computer Science*, pages 343–355. Springer, 2004.


Jacques Patarin.
Luby-Rackoff: 7 Rounds Are Enough for $2^{n(1-\epsilon)}$ Security.
In Dan Boneh, editor, *Advances in Cryptology - CRYPTO 2003, 23rd Annual International Cryptology Conference, Santa Barbara, California, USA, August 17-21, 2003, Proceedings*, volume 2729 of *Lecture Notes in Computer Science*, pages 513–529. Springer, 2003.

Jacques Patarin.

A Proof of Security in $O(2^n)$ for the Xor of Two Random Permutations.

In Reihaneh Safavi-Naini, editor, *Information Theoretic Security, Third International Conference, ICITS 2008, Calgary, Canada, August 10-13, 2008, Proceedings*, volume 5155 of *Lecture Notes in Computer Science*, pages 232–248. Springer, 2008.